1.Interest Rates and Time Value of Money

With at compound interest, the value today is *present value (PV)*, and the value n periods from today is the *future value (FV)*. If funds are invested at a periodic compound interest rate i for n periods, the basic relationships are:

$$FV = PV(1+i)^n \qquad PV = \frac{FV}{(1+i)^n}$$

In many instances where paments are made for a period less than a year (e.g., monthly, quarterly), the period interest rate is stated as a *nominal annual rate*, which is the interest rate per period multiplied by the number of periods per year.

In the general case for which there are m payment periods per year, we denote the nominal rate by $i^{(m)}$. The periodic interest rate is $\frac{i^{(m)}}{m}$, and the effefctive rate is:

$$i = \left(1 + \frac{i^{(m)}}{m}\right)^m - 1$$

And,

$$i^{(m)} = m\left[(1+i)^{\frac{1}{m}} - 1\right]$$

The rate of discount d, is used extensively in actuarial mathematics.

$$d = 1 - \frac{1}{(1+i)} = \frac{i}{(1+i)}$$

Some Essential Interest Theory Notation

$v = \frac{1}{1+i}$ $d = iv$ $d = 1 - v$ $i - d = id$

Nominal rates of discount,

$$1 - d = \left(1 - \frac{d^{(m)}}{m}\right)^m$$

Continuous Interest and Force of Interest

Interest may also be paid on a continuous basis. The constant continuous growth rate is denoted by δ. In general, for the compound growth model $a(t) = (1+i)^t$, we can write

$$a(t) = e^{\delta t} \qquad \delta = ln(1+i)$$

$$(1+i)^n = e^{\delta n}$$

$$v^n = (1+i)^{-n} = e^{-\delta n}$$

Exercies

1. Using compound interest rate, investment amount and annual period information as inputs; Write the function that calculates the cumulative value (future value) at the end of the period.

```r
Future_value<- function(i,I,n){
  FV=I*(1+i)^n
  return(FV)
}
Future_value(i=0.05,1000,4)
```

```
## [1] 1215.506
```

2. Write the function that calculates future value using investment value and time inputs, which are valued as compound and continues interest rates (as function $\delta_t = \frac{2}{t+1}$), respectively.

```r
Present_value<-function(i,delta,n1,n2,I){
FV1<-I*(1+i)^n1
func<-function (t)(2/(t+1))
FV2<-integrate(func,lower=n1,upper = n2)
FV<-FV1*exp(FV2$value)
return(FV)
}
Present_value(0.05,(2/(t+1)),2,5,5000)
```

```
## [1] 22050
```

3. Find effective interest rate for given nominal interest rate.

```r
Eff<-function(im,m){
  i<-(1+(im/m))^m-1
  return(i)
}
Eff(0.07,4)
```

```
## [1] 0.07185903
```